13.2 Complements and Unions of Events

A sample space is a set.
An event is a set.

The idea of this section is to combine the ideas of sets with probability.

We will use complements, unions, and intersections.

Recall:
The complement of a set is the collection of elements not in that set.
A' = { elements not in A }

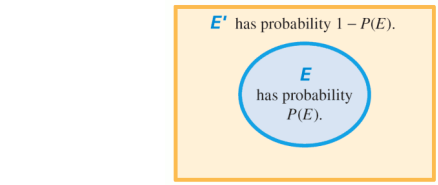
The complement of an event E, is the collection of outcomes not in E.
E' = { outcomes not in E }

If an outcome is in the sample space, it must be in E or E'.

So E and E' give all outcomes.

So P(E) + P(E') = 1 (100%)

COMPUTING THE PROBABILITY OF THE COMPLEMENT OF AN EVENT If E is an event, then $P(E') = 1 - P(E)$.



A drug was administered.

The probability that the person got better was 0.28. (28%)

What is the probability that the person did not get better?

A drug was administered.

The probability that the person got better was 0.28. (28%)

What is the probability that the person did not get better?

P(E') = 1 – P(E)
= 1 – 0.28
= 0.72

A single card is removed from a deck.

What is the probability that it is not the Jack of Clubs?

A single card is removed from a deck.

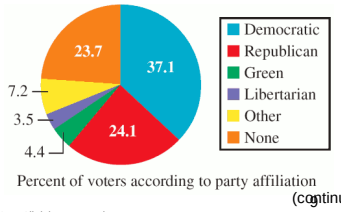
What is the probability that it is not the Jack of Clubs?

E = Jack of Clubs
P(E) = 1/52

E' = not Jack of Clubs
P(E') = 1 – 1/52 = 52/52 – 1/52 = 51/52

Complements of Events

• Example: The graph shows the party affiliation of a group of voters. If we randomly select a person from this group, what is the probability that the person has a party affiliation?



Percent of voters according to party affiliation

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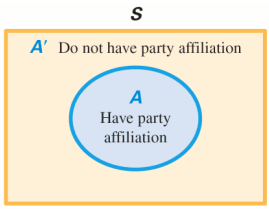
Complements of Events

• Solution: Let *A* be the event that the person we select has some party affiliation. It is simpler to calculate the probability of *A′*. Since 23.7% have no party affiliation,



Complements of Events

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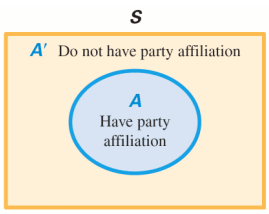


$$P(A) = 1 - P(A') = 1 - 0.237 = 0.763.$$

Word problems:

OR means union
- Heart or Ace
- Heart U Ace

AND means intersection
- Heart and Ace
- Heart ∩ Ace

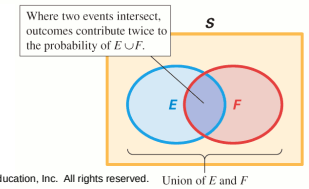
Unions of Events

RULE FOR COMPUTING THE PROBABILITY OF A UNION OF TWO EVENTS If E and F are events, then

$$P(E \cup F) = P(E) + P(F) - P(E \cap F).$$

If E and F have no outcomes in common, they are called *mutually exclusive events*. In this case, because $E \cap F = \varnothing$, the preceding formula simplifies to

$$P(E \cup F) = P(E) + P(F).$$



One card was drawn from a deck.

What is the probability that it was a Heart or an Ace?

One card was drawn from a deck.

What is the probability that it was a Heart or an Ace?

P(Heart) = 13/52

P(Ace) = 4/52

P(Heart ∩ Ace) = 1/52 only "Ace of Hearts"

One card was drawn from a deck.

What is the probability that it was a Heart or an Ace?

P(Heart) = 13/52

P(Ace) = 4/52

P(Heart ∩ Ace) = 1/52

P(Heart U Ace) = P(Heart) + P(Ace)
- P(Heart ∩ Ace)
= 16/52 = 4/13

Unions of Events

- Example: If we select a single card from a standard 52-card deck, what is the probability that we draw either a heart or a face card?
- Solution: Let H be the event "draw a heart" and F be the event "draw a face card." We are looking for $P(H \cup F)$.

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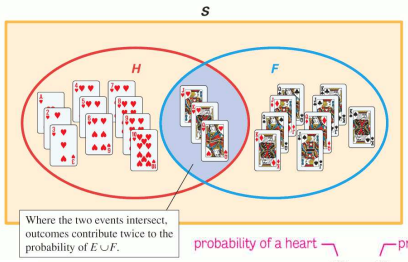
Unions of Events



There are 13 hearts, 12 face cards, and 3 cards that are both hearts and face cards.

probability of a heart — probability of a face card

$$P(H \cup F) = P(H) + P(F) - P(H \cap F) = \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{22}{52} = \frac{11}{26}$$

probability of a heart that is a face card

If you are given 3 out of the 4 terms in the equation

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

Then you can use algebra to find the remaining term.

This can also be read as

P(E or F) = P(E) + P(F) – P(E and F)

The probability a UT student is
- an Education major is 0.09.
- an Ed. major and in athletics is 0.01
- in Ed or athletics is 0.12.

What is the probability that a UT student is in athletics?

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What is the probability that a UT student is in athletics?

P(Ed or ath) = P(Ed) + P(ath) – P(Ed or ath)
0.12 = 0.09 + P(ath) – 0.01
0.12 = 0.08 + P(ath)
0.04 = P(ath)

The probability of a person being
- happy is 0.45
- a millionaire is 0.02
- happy or a millionaire is 0.46

What is the probability that a person is happy and a millionaire?

The probability of a person being
- happy is 0.45
- a millionaire is 0.02
- happy or a millionaire is 0.46

What is the probability that a person is happy and a millionaire?

P(happy or $$$) =
P(happy) + P($$$) - P(happy and $$$)
0.46 = 0.45 + 0.02 – P(H and $$$)
0.46 = 0.47 – P(H and $$$)
P(H and $$$) = 0.01